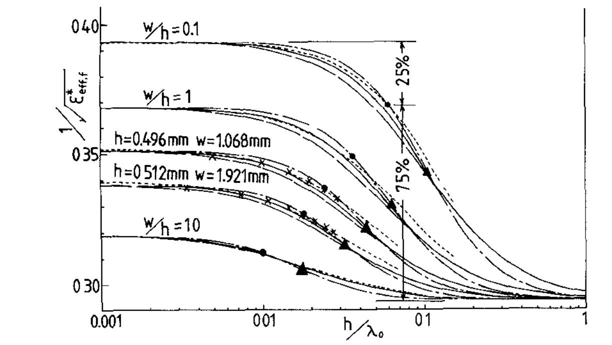

Fig. 3. The $1/\sqrt{\epsilon^*_{\text{eff},f}}$ for the microstrip line on sapphire substrate. $\epsilon^*_\xi = 9.4, \epsilon^*_\eta = 11.6, \gamma = 0$ in Fig 1 —·— formula $K(4)$, — — formula $Y(8)$, —— formula $YK(10)$, ------ theoretical results; [4] for $w/h = 0.1$, 1, 10, and [5] for the other two dotted lines "x" experimental results [3], ● $f_i(11)$, ▲ $f_K(5)$

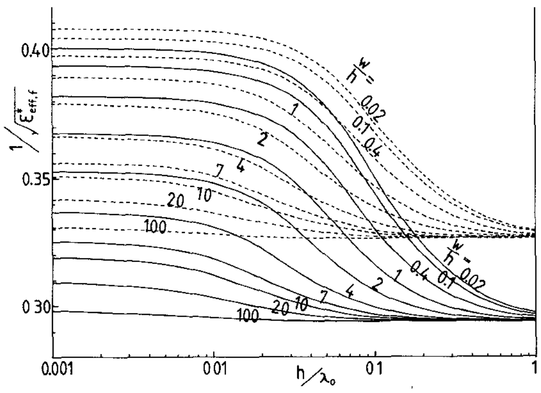

Fig. 4. The $1/\sqrt{\epsilon^*_{\text{eff},f}}$ for the microstrip line on sapphire substrate calculated by formula $YK(10)$. —— $\epsilon^*_x = 9.4, \epsilon^*_y = 11.6$; ------ $\epsilon^*_x = 11.6, \epsilon^*_y = 9.4$.

shows good agreement with the theoretical and experimental results and provides good design data. Fig. 4 shows the numerical results of formula YK. The equations also show the influence of the cutting angle of substrate on the dispersion properties.

IV. Conclusion

Three simple approximate dispersion formulas (K, Y, YK) have been derived. The results obtained have been compared with other available results with good agreement. We have found that formula YK gives good design data.

The author thanks Prof. Naito, Tokyo Institute of Technology, for giving the copy of [8].

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Important Role of Inflection Frequency in the Dispersive Property of Microstrip Lines

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Abstract —**It is represented that the inflection frequency f_i has the important role in the dispersive property of microstrip lines. This f_i is related to the coupling between the TEM mode and the TM_0 mode. Using f_i, an approximate dispersion formula is derived by improving Schneider's formula. The results obtained by the present formula are compared with the other available results; good agreement is seen.**

A microstrip transmission line is an essential part of an integrated circuit which is a fundamental component in modern microwave devices. With its increasing use at higher frequencies, a number of workers have theoretically studied the dispersive properties of microstrip lines [1]–[5] (good bibliographies are given in [6] and [10]). However, their analyses usually require a complicated computer program and, in some cases, enormous computing time. Recently, their results have been compared [6]. On the other hand, the computer-aided design of microstrip circuits requires accurate and reliable information on the dispersive behavior. A few approximate equations satisfying these requirements have been formulated [8]–[15], [17].

In a microstrip geometry, transverse TM- and TE-wave modes exist. Only even-order TM surface-wave modes and odd-order TE surface-wave modes are possible [18]. The TM_0 mode is dominant since it has a zero frequency cutoff [18], while higher order

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modes have cutoff frequencies beyond the range of interest, therefore, having a negligible effect [9]. Therefore, the important modal coupling is that between the TEM microstrip-line mode and the TM_0 mode [7], [9], [16]. In this short paper, the inflection frequency f_t is related to this coupling. An approximate dispersion formula is derived by using this f_t.

We consider the microstrip line of strip width w, substrate thickness h, and permittivity of substrate $\epsilon = \epsilon^* \epsilon_0$ (ϵ^* is the relative dielectric constant, ϵ_0 is the permittivity of vacuum). Using the wave equations and the continuity conditions for fields at the interface of the different media in such a line [18], we can derive a coupling frequency $f_{K,\mathrm{TM}}$ defined as that frequency at which the phase velocity of the TEM mode equals the phase velocity of the TM_0 mode, and a coupling frequency $f_{K,\mathrm{TE}}$ between the TEM mode and the TE_1 mode

$$f_{K,\mathrm{TM}} = \frac{v_0 \tan^{-1}\left(\epsilon^* \sqrt{\dfrac{\epsilon^*_{\mathrm{eff},0} - 1}{\epsilon^* - \epsilon^*_{\mathrm{eff},0}}}\right)}{2\pi h \sqrt{\epsilon^* - \epsilon^*_{\mathrm{eff},0}}} \tag{1}$$

$$f_{K,\mathrm{TE}} = \frac{v_0 \left(\dfrac{\pi}{2} + \tan^{-1}\sqrt{\dfrac{\epsilon_{\mathrm{eff},0} - 1}{\epsilon^* - \epsilon^*_{\mathrm{eff},0}}}\right)}{2\pi h \sqrt{\epsilon^* - \epsilon^*_{\mathrm{eff},0}}} \tag{2}$$

where $\epsilon^*_{\mathrm{eff},0}$ denotes the effective dielectric constant at zero frequency and is obtained also by

$$\epsilon^*_{\mathrm{eff},0} = 1 + q(\epsilon^* - 1) \tag{3}$$

where q is the effective filling fraction which has been tabulated for a wide range of ϵ^* and w/h [20], and v_0 denotes the speed of light in free space. Accurate approximate formulas have been obtained [19], [21]. We have the following limits for $f_{K,\mathrm{TM}}, f_{K,\mathrm{TE}}$ since q lies between 0.5 (when $w/h \to 0$) and 1 (when $w/h \to \infty$) [19], [20], [21]

$$1.5(q = 0.5, \epsilon^* \to \infty) \leqslant \frac{f_{K,\mathrm{TE}}}{f_{K,\mathrm{TM}}} \leqslant 3(q = 1). \tag{4}$$

We also note that the TM_0 mode is dominant since $f_{K,\mathrm{TM}}$ is always smaller than $f_{K,\mathrm{TE}}$.

Schneider [10] showed the four properties which have to be satisfied by the normalized phase velocity

$$\tilde{v}_p = \frac{v}{v_0} = \frac{1}{\sqrt{\epsilon^*_{\mathrm{eff},f}}}. \tag{5}$$

They are as follows: 1) $\tilde{v}_p$ is a monotonically decreasing function of f; 2) $\tilde{v}_p = 1/\sqrt{\epsilon^*_{\mathrm{eff},0}}$, $\partial \tilde{v}_p / \partial f = 0$ when $f = 0$; 3) $\tilde{v}_p \to 1/\sqrt{\epsilon^*}$, $\partial \tilde{v}_p/\partial f \to 0$ when $f \to \infty$; 4) $\partial^2 \tilde{v}_p / \partial f^2 = 0$ in the vicinity of the cutoff frequency $f_{C,\mathrm{TE}}$ of the TE_1 surface wave. They have been checked on the available data obtained theoretically or experimentally by many workers. The first three properties have been confirmed. However, the last property could not be confirmed in all cases. The curve of this dispersion formula which is obtained by replacing f_K with $f_{C,\mathrm{TE}}$ in (8) and (9) has an inflection frequency f_t as follows:

$$f_t = f_{C,\mathrm{TE}} / \sqrt{3} \tag{6}$$

where

$$f_{C,\mathrm{TE}} = v_0 / \left(4h\sqrt{\epsilon^* - 1}\right). \tag{7}$$

We notice in (7) that $f_{C,\mathrm{TE}}$ is a function of only h and ϵ^*, but not of w. Therefore, f_t is also independent of w. On the other hand,

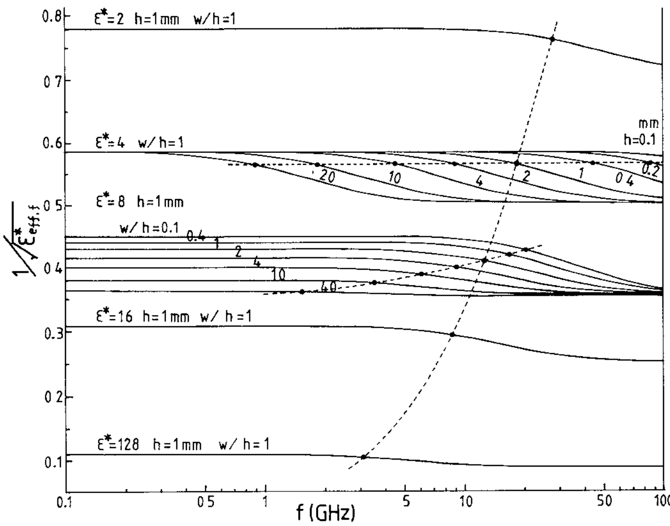

Fig. 1. Dispersion behavior of the approximate formula (8) for some variations of parameters ϵ^*, h, and w/h.

we have observed from available data that f_t must be a function of not only h and ϵ^*, but also w. However, the author was interested in Schneider's formula with the simple form and considered improving it.

The order of magnitude of $\epsilon^*_{\mathrm{eff},f}$ is determined by $\epsilon^*_{\mathrm{eff},0}$, that is, w/h and ϵ^* (see Fig. 1). On the other hand, f_t strongly influences the dispersive nature of the line (see Fig. 1, • denotes f_t). Therefore, the value of f_t is very important in deriving the approximate formula. The author believes that f_t ought to have a strong relation to $f_{K,\mathrm{TM}}$ shown in (1) since the TM_0 mode is dominant. The author proposes the following approximate formula based upon the theoretical considerations given above:

$$\frac{1}{\sqrt{\epsilon^*_{\mathrm{eff},f}}} = \frac{\dfrac{1}{\sqrt{\epsilon^*}}\left(\dfrac{f}{f_K}\right)^2 + \dfrac{1}{\sqrt{\epsilon^*_{\mathrm{eff},0}}}}{\left(\dfrac{f}{f_K}\right)^2 + 1} \tag{8}$$

where

$$f_K = f_{K,\mathrm{TM}} \Big/ \left(1 + \frac{w}{h}\right) \tag{9}$$

and the denominator in (9) is only one corrective factor. Schneider's formula [10] is identical to the one obtained by replacing f_K with $f_{C,\mathrm{TE}}$ in (8).

The dispersion curve in (8) has the following inflection frequency f_t at which the second-order derivative of $1/\sqrt{\epsilon^*_{\mathrm{eff},f}}$ with respect of f is zero:

$$f_t = f_K / \sqrt{3}. \tag{10}$$

The $1/\sqrt{\epsilon^*_{\mathrm{eff},f}}$ at f_t has, for any ϵ^*, h, and w, the following value:

$$\frac{1}{\sqrt{\epsilon^*_{\mathrm{eff},f_t}}} = \frac{1}{4}\left(\frac{3}{\sqrt{\epsilon^*_{\mathrm{eff},0}}} + \frac{1}{\sqrt{\epsilon^*}}\right). \tag{11}$$

Fig. 1 shows the dispersive behavior of the present formula (8) for several variations of the parameters (ϵ^*, h, and w/h). The dispersion curves are monotonically decreasing $1/\sqrt{\epsilon^*_{\mathrm{eff},0}}$ (at $f = 0$) to $1/\sqrt{\epsilon^*}$ (at $f = \infty$), due to an increase in frequency. On these curves, f_t is the point fallen down just a quarter of all variation

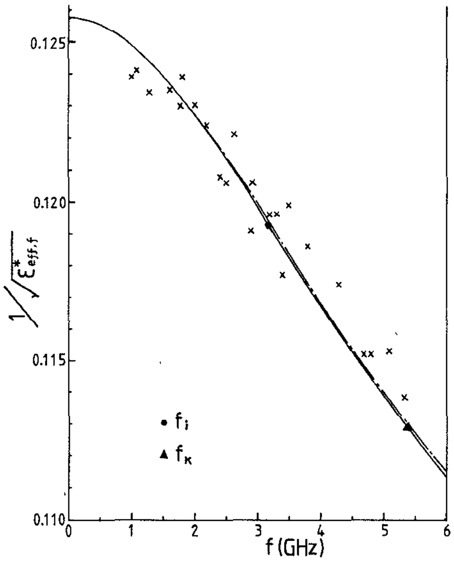

Fig. 2. Dispersion behavior of $1/\sqrt{\epsilon^*_{eff,f}}$. $\epsilon^* = 104$, $h = 1.27$ mm, $w/h = 0.6$, $q = 0.604191$, $f_t = 3.3186$, $f_K = 5.7480$, $f_{K,TM} = 9.1968$, $f_{C,TE} = 5.8148$ GHz. —— the present formula, —-— Schneider's formula [10], × Hartwig *et al.* [16].

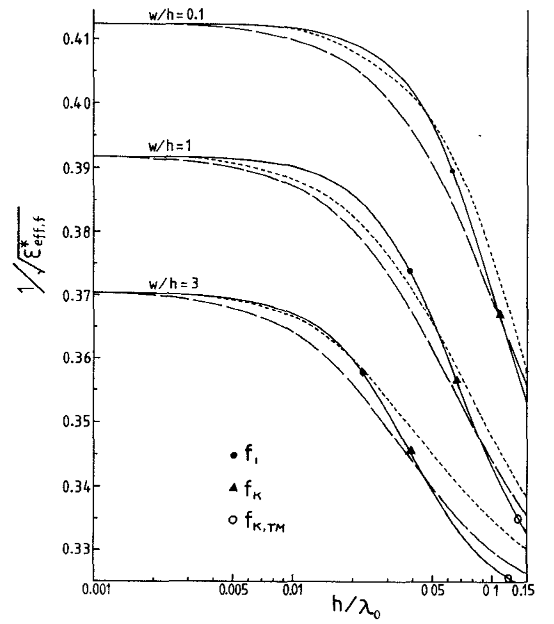

Fig. 3. Dispersion behavior of $1/\sqrt{\epsilon^*_{eff,f}}$. $\epsilon^* = 9.7$; $q = 0.560246$, $f_t = 18.945$, $f_K = 32.814$, $f_{K,TM} = 36.095$, $f_{C,TE} = 25.409$ GHz for $w/h = 0.1$; $q = 0.633505$, $f_t = 11.512$, $f_K = 19.940$, $f_{K,TM} = 39.881$, $f_{C,TE} = 25.409$ GHz for $w/h = 1$; $q = 0.722898$, $f_t = 6.6845$, $f_K = 11.577$, $f_{K,TM} = 46.311$, $f_{C,TE} = 25.409$ GHz for $w/h = 3$. $h/\lambda_0 = f(\text{GHz})/299.7925$. —— the present formula (8), —— Yamashita's formula [14], ------- Kowalski *et al.* [1].

from the $1/\sqrt{\epsilon^*_{eff,0}}$. Therefore, we can approximately draw the curves of $1/\sqrt{\epsilon^*_{eff,f}}$ versus f if $1/\sqrt{\epsilon^*_{eff,0}}$, $1/\sqrt{\epsilon^*}$, and f_t are given. We can graphically confirm in Fig. 1 the important results; the larger is ϵ^*, w/h, or h, the lower is the inflection frequency f_t. Thus, we can understand the important roles of f_t.

Figs. 2 and 3 compare the results of the present formula with the other available results [1], [10], [14], [16]. The values of $1/\sqrt{\epsilon^*_{eff,0}}$ at the zero frequency were calculated by the Green's function technique [20] which has a very high degree of accuracy. The good agreement of this formula (solid line) and Schneider's formula (dot-dash-line) [10] as shown in Fig. 2 is due to the good agreement of $f_K = 5.748$(GHz) and $f_{C,TE} = 5.814$(GHz) that is, both the inflection frequencies f_t are nearly equal. In Fig. 3, the dotted lines denotes the theoretical results by Kowalski *et al.* [1], the broken lines denotes the results of Yamashita's formula [4], and the solid lines the results of the present formula. Good agreement can be seen. We find that the present formula has good agreement with the theoretical results near f_t.

In summary, the important roles of f_t are represented by letting f_t be related to the coupling between the TEM_L mode and the TM_0 mode $f_{K,TM}$. This result is applied to the article [22].

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